Think back to the Tangent Line Problem introduced in sect. 1.1.
Illustration:

The slope of the secant line on $f$ between $x$ and $x+\Delta x$ :

This is the $\qquad$ rate of change for $f$ on the interval $[x, x+\Delta x]$.

The slope of the tangent line on $f$ at $x$ :

This is the $\qquad$ rate of change for $f$ at $x$.
*This is given a special name: the $\qquad$ of $f$.

Notation:

General form:
At a specific point $x=c$ :

To find the instantaneous rate of change ( $\qquad$ ), we need to compute this limit, either in general, or at a specific $x$ value.

Math 250 - Sect. 2.1: The Derivative
-example- Find the slope of the function $f(x)=3 x-4$ at any point $(x, f(x))$
-example- a. Find the slope of the function $f(x)=x^{2}-2 x$ at any point $(x, f(x))$
b. *Now, find the slope of the tangent line at $\boldsymbol{x}=\mathbf{3}$
c. Write the EQUATION of the tangent line at that point.
d. Determine the value(s) of x for which the function would have a horizontal tangent line.

Math 250 - Sect. 2.1: The Derivative
-example- Find the equation of the line tangent to the curve $f(x)=\frac{1}{x+2}$ when $x=1$.

SKETCH the graph and draw in the tangent line:
II. Differentiability. A function is said to be differentiable at a point $x=c$ if the derivative exists at that point. Since the derivative is defined as a limit, then both the right and left hand limits would have to be the SAME for the derivative to exist.
*A function is NOT differentiable at any point $x=c$ if:

1. It is not CONTINUOUS at that $x$ value.

$$
\text { -example- } \quad f(x)=\frac{1}{x-2}
$$

*NOTE: Differentiability implies continuity. The reverse is not true.

Math 250 - Sect. 2.1: The Derivative
2. The curve becomes VERTICAL at that $x$ value.
-example- $\quad f(x)=x^{1 / 3}$
*NDERIV feature on Calc:
3. The curve has a SHARP POINT at that $x$ value.
-example- $f(x)=|x+1|+2$
*A function is differentiable at a point if it has local linearity.

